Calculating the prolonged length of service time through shock model in an organization

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Abstract

Every organization needs a workforce suitable for its tasks in order to reach its business aims. In an organization wastages are seen when employees moving from one grade to another, are exposed to different factors influencing them to leave the organization. The threshold level is the maximum amount of wastage that can be permitted in the organization beyond which the organization reaches a point of breakdown. This paper is an attempt to determine the expected time for recruitment, assuming the threshold distribution as Generalized Rayleigh distribution introduced by Surles and Padgett 2001[1].

Keywords: Expected time; Organization; Threshold; Wastage

Introduction

Manpower planning can be defined as an attempt to match the supply of people with the jobs available in an organization, with the aid of statistical techniques. Length of service in a grade should necessarily be a natural criterion for promotion in order to create a healthy atmosphere among the employees. The leaving pattern or wastage process in any organization is also termed complete length of service. Mathematical model is obtained for the expected time of breakdown point to reach the threshold level.

Bartholomew 1971[2] provided a general review of the application of stochastic modeling to social systems, while Bartholomew and Forbes 1979[3] developed a more specific application of these principals to the manpower planning problem. Esary et al.1973[4] discussed that any component or device when exposed to shocks which cause damage to the device, or system, is likely to fail when the total accumulated damage exceeds a level called the threshold. The rate of accumulation of damage determines the lifetime of the component or device. One can see for more detail for the threshold to attain the expected time in Pandiyan et al. (2010)[5], Jeeva et al. (2004)[6] and Sathiyamoorthi (1980)[7].

Assumptions

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand.

- Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives, and promotions are made.
- The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter- arrival times between successive occasions of wastage are i.i.d. random variables.

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• If the total depletion exceeds a threshold level Y, which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.

• The process which generates the exits, the sequence of depletions and the threshold are mutually independent.

Notations

X_i : A continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i = 1,2,\dots.k$ and X_i's are i.i.d and $X_i = X$ for all i.

Y : a continuous random variable denoting the threshold level having Generalized Rayleigh distribution.

$g(.)$: The probability density functions of X.

$g_k(.)$: The k- fold convolution of i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

T : a continuous random variable denoting time to breakdown of the system

$g^*(.)$: Laplace transform of.

$g_k^*(.)$: Laplace transform of $g_k(.)$.

$h(.)$: The p.d.f. of random threshold level which has Generalized Rayleigh distribution and $H(.)$ is the corresponding c.d.f.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: P.d.f. of random variable U with corresponding c.d.f. $F(.)$.

$F_k(t)$: The k-fold convolution functions of $F(.)$.

$S(.)$: The survivor function i.e. $P[T>t]$.

$L(t)$: $1-S(t)$

$V_k(t)$: Probability that there are exactly 'k' policies decisions in (O, t).

Results

In this paper, having the threshold which follows Generalized Rayleigh distribution is discussed with the shape parameter considered. The expected time and variance are obtained. The two-parameter Generalized Rayleigh distribution is a particular member of the Generalized Weibull distribution, originally proposed by Mudholkar and Srivastava (1993)[8].

Let Y be the random variable which has the cdf defined as

$$F(x:\alpha,\lambda) = \left(1 - e^{-(\lambda x)^2}\right)^{\alpha}$$

The corresponding survival function is

$$\left(1 - e^{-2\lambda x}\right)^{\alpha} \sim \left(1 - e^{-2\lambda x}\right)^{n} = \sum_{r=0}^{n} (-1)^r \binom{n}{r} \left(e^{-(\lambda x)^2}\right)^r$$

$$\bar{H}(x) = \sum_{r=1}^{n} (-1)^{r+1} \binom{n}{r} \left(e^{-(\lambda x)^2}\right)^r \quad (1)$$

There may be no practical way to inspect an individual item to determine its threshold y. In this case the threshold must be a random variable. The shock survival probability is given by

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^{\infty} g_k(x)\, \bar{H}(x)\, dx \quad (2)$$

$$= \int_0^{\infty} g_k(x) \left[\sum_{r=1}^{n} (-1)^{r+1} \binom{n}{r} \left(e^{-(\lambda x)^2}\right)^r\right] dx$$

$$= \sum_{r=1}^{n} (-1)^{r+1} \binom{n}{r} g_k^*(\lambda r)^2 \quad (3)$$

Equation denotes the k^{th} convolution.

Therefore $S(t)=P[T>t]$ is the survival function which gives the probability that the cumulative will fail only after time t.

$S(t)=P[T>t]=$ Probability that the total damage survives beyond t

$$= \sum_{k=0}^{\infty} P\ \{\text{there are exactly k contacts in } (0,t]\ *\ P\ (\text{the total cumulative } (0,t]\}$$

$$S(t) = P(T>t) = \sum_{k=0}^{\infty} V_k(t) P(X_i < Y) \quad (4)$$

A renewal process is a counting process such that the time until the first event occurs has some distribution F, the time between the first and second event has, independently of the time of the first event, the same distribution F, and so on. When an event occurs we say that a renewal has taken place. It may happen that successive shocks become increasingly effective in causing damage, even though they are independent. This means that, the distribution function of the damage is

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \qquad \text{with} \quad F_0(t) = 1$$

$$= \sum_{k=0}^{\infty}\sum_{r=1}^{n}[F_k(t) - F_{k+1}(t)]\binom{n}{r}(-1)^{r+1}g_k^*(\lambda r)^2$$

$$= \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1} - \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}(1-g_k^*(\lambda r)^2)\sum_{k=1}^{\infty}[F_k(t)][g^*(\lambda r)^2]^{k-1}$$

$P(T < t) = L(t)$ = The distribution functions of life time (T). $L(t) = 1 - S(t)$

$$= 1 - \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1} - \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}(1-g^*(\lambda r)^2)\sum_{k=1}^{\infty}[F_k(t)][g^*(\lambda r)^2]^{k-1}$$

Where $[f^*(s)]^k$ is Laplace transform of $V_k(t)$ since the inter-arrival times are i.i.d. The above equation can be rewritten as,

$$= \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}(1-g^*(\lambda r)^2)\sum_{k=1}^{\infty}[F_k(t)][g^*(\lambda r)^2]^{k-1} \tag{5}$$

$$E(T) = -\frac{d}{ds}L^*(s) \quad \text{given} \quad s = 0$$

$$E(T^2) = \frac{d^2L^*(s)}{ds^2} \quad \text{given} \quad s = 0$$

decreasing in for each t. It is also known from renewal process that

From which the Variance can be obtained.

$$f^*(s) = \left(\frac{c}{c+s}\right)$$

$$l^*(s) = \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\frac{(1-g^*(\lambda r)^2)f^*(s)}{(1-g^*(\lambda r)^2 f^*(s))} \tag{6}$$

$$= \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\frac{c(1-g^*(\lambda r)^2)}{(c+s-g^*(\lambda r)^2 c)} \tag{7}$$

The mean and variance of the time to threshold to cross the breakdown point is derived.

$$E(T) = \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\frac{1}{c(1-g^*(\lambda r)^2)} \qquad \text{on simplification} \tag{8}$$

$$E(T^2) = \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\frac{2}{c^2(1-g^*(\lambda r)^2)^2} \qquad \text{on simplification} \tag{9}$$

$$V(T) = E(T^2) - [E(T)]^2$$

The inter-arrival time of the threshold follows exponential distribution. The Laplace transformation of the exponential is given by $\left[{}^{\mu}/_{\mu+\lambda}\right]$.

$$g^*(.)\sim exp(\mu), \quad g^*(\lambda r)^2 = \left[\frac{\mu}{\mu+(\lambda r)^2}\right]$$

$$E(T) = \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\left(\frac{1}{c}\right)\left[\frac{(\mu+(\lambda r)^2)}{(\lambda r)^2}\right] \tag{10}$$

$$E(T^2) = \sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\left(\frac{2}{c^2}\right)\left[\frac{(\mu+(\lambda r)^2)}{(\lambda r)^2}\right]^2$$

$$V(T) = \left[\sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\left(\frac{2}{c^2}\right)\left[\frac{(\mu+(\lambda r)^2)}{(\lambda r)^2}\right]^2\right] - \left[\sum_{r=1}^{n}\binom{n}{r}(-1)^{r+1}\left(\frac{1}{c}\right)\left[\frac{(\mu+(\lambda r)^2)}{(\lambda r)^2}\right]\right]^2 \tag{11}$$

Special case $(\alpha = 1)$

Let the random variable denoting inter arrival time which follows exponential with parameter. Now , substituting in the below equation we get,

$$E(T) = \frac{(\mu+\lambda)^2}{c(\lambda^2 - 2\mu\lambda)} \tag{12}$$

$$V(T) = \frac{(\mu+\lambda)^4}{c^2[\lambda^2 - 2\mu\lambda]^2} \tag{13}$$

The shape parameter of the Generalized Rayleigh distribution α is kept fixed i.e. $\alpha = 1$. We obtained the following equation (12) and (13) as the expected time to E (T) and variance V (T).

Numerical Illustration

Simulation models are particularly useful in studying small systems where random fluctuations are likely to be more serious. To illustrate the method described in this paper (special case), we gave some limited simulation results. The expected time and variance from the above equation and was found with the changes in parameters and with increasing parameters which is observed in the given figures below.

Conclusion

When μ is kept fixed, the inter-arrival time 'c' which follows exponential distribution is an increasing case by the process of renewal theory. Therefore, the value of the expected time *E (T)* to cross the breakdown point of threshold is found to be decreasing, in all the cases of the parameter value μ=0,5,1,1,5,2. When the value of the parameter increases, the expected time is also found decreasing, as observed in Figure 1. The same case is found in Variance V (T) which is observed in Figure 3.

When λ is kept fixed and the inter-arrival time 'c' increases, the value of the expected time *E (T)* to cross the threshold is found to be decreasing, in all the cases of the parameter

Fig 1.

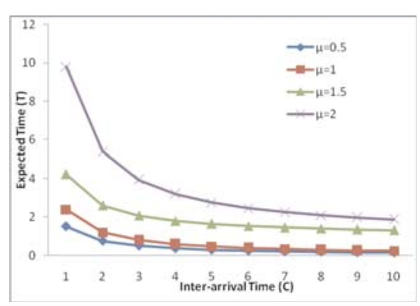


Fig 2.

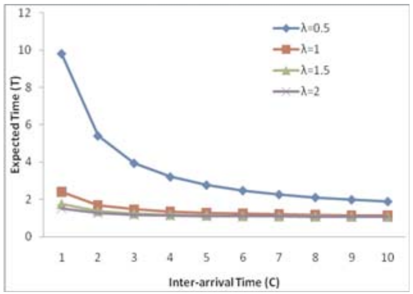


Fig 3.

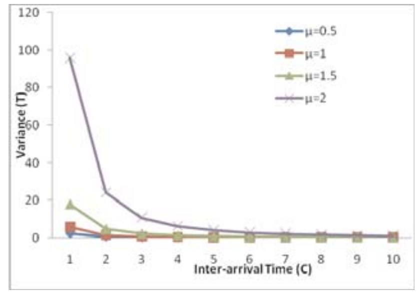
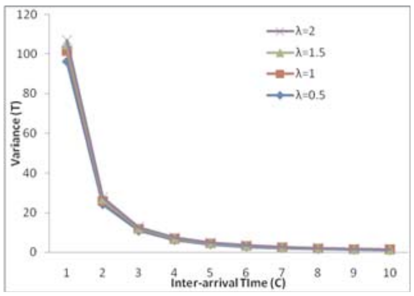


Fig 4.



value λ=0,5,1,1,5,2. When the value of the parameter increases, the expected time is found increasing, as indicated in Figure 2. The same case is observed in the breakdown point of threshold for Variance *V (T)* which is observed in Figure 4.

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